Table 1 Table for T_∞ for different values of λ

ρ	u_0	I_0, deg	λ	μ_∞	T_∞
1.13×10^{-3}	1.539	313	3×10^{-3}	0.34×10^3	264°2
1.13×10^{-3}	1.5393	313	1.6×10^{-4}	1.47×10^3	264°1
1.13×10^{-3}	1.53934	313	3×10^{-5}	3.6×10^3	258°0

For air, $\mu = 1.64 \times 10^{-4}$, $\beta = \frac{1}{273}$, $C_v = 0.1689$, $C_p = 0.2375$, $\gamma = 1.66$, $\rho = 1.13 \times 10^{-3}$ approximately at 40°C., and $g = 981$; therefore, $\alpha = 1.74 \times 10^{-3}$. Since m should be $< \alpha$, i.e., $u_0 < 1.53935$ cm/sec., then $\lambda = 3.15(1.53935\text{-}u_0)$; since a small value of λ is desirable, u_0 should be very close to 1.53935, but should not exceed it. Therefore, the initial motion at $x = 0$ admissible under this solution is very slow. For a different value of u_0, T_0 and $\rho_0 = 1.3 \times 10^{-3}$. The values of u_∞ and T_∞ are given in Table 1.

Now $T^* = [(T - T_\infty)/(T_0 - T_\infty)]$, and T^* is plotted against x for different values of λ in case I, case II, and case III (see Fig. 1).

At higher temperature, ρ is much smaller, and we can take u_0 greater than that in the preceding case. Also, if we consider a column of gas on a planet or a star where g is large and ρ is small, u_0 can be taken much larger.

References

[1] Morduchow, M. and Libby, P. A., "On a complete solution of the one-dimensional flow equations of a viscous, heat-conducting compressible fluid," J. Aeronaut. Sci. **16**, 674–684 (1949).

[2] Pai, S. I., *Viscous Flow Theory—I—Laminar Flow* (D. Van Nostrand Co., Inc., New York, 1956).

[3] Ludford, G. S. S., "The classification of one-dimensional flows and the general shock problem of a compressible, viscous, heat conducting fluid," J. Aeronaut. Sci. **18**, 830–834 (1951).

Delays in Initiation of Discharges in Pulsed Plasma Accelerators

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PULSED plasma accelerators used in electrical propulsion investigations are operated by the charging of a capacitor bank, and then rapidly discharging the bank through a propellant gas that is contained between a pair of coaxial or nozzle shaped electrodes. The gas, which is initially un-ionized, is quickly converted by the discharge to a highly ionized plasma and is ejected with high velocity from the accelerator by MHD forces. In some of these devices[1, 2] the capacitor bank is permanently connected to the electrodes. The accelerators operate in a vacuum chamber that is maintained at sufficiently low pressure so that no discharge occurs between the electrodes while the bank is charging. When the bank is fully charged, a puff of propellant is injected between the electrodes by very briefly opening a gas valve, which raises the pressure sufficiently to initiate the discharge. The efficiency of propellant utilization in these accelerators can be markedly affected by the time interval between the injection of the gas and the time when the discharge current becomes large enough to accelerate the gas. If too long a time elapses, propellant may leak out of the accelerator into the surrounding vacuum and be wasted. An interval of even a few hundred microseconds may be sufficient to cause appreciable loss.

It is often assumed that the discharge will commence as soon as the pressure in a region between the electrodes is raised by the puff of gas so that the d.c. sparking potential in that region becomes equal to or slightly less than the interelectrode voltage. The subsequent rate of current rise is considered to be determined solely by the circuit inductance and capacitance. These assumptions neglect several factors that could lengthen the time required to reach a high current level. Among these are statistical time lag,[3] which is the time required for a free electron to enter the gas. Since the propellant is initially un-ionized, it will remain a practically perfect insulator until at least one free electron enters it. The ionizing collisions made by this electron produce additional charge carriers that initiate the electrical breakdown. Statistical lags can be relatively long, depending on the strength of the sources supplying free electrons to the discharge region. Generally, radioactive or other sources of free electrons are not used in pulsed plasma accelerators, and the free electron must originate in an accidental manner.

After initiation of breakdown, current density grows to large values, densities exceeding 10^7 amp/m^2 being common.[2] To support these large densities, the cathode must emit electrons copiously. It may take an appreciable time after initial breakdown for the physical mechanisms that provide this emission to become operative. This time would be an additional source of delay before current could grow to large magnitudes.

To investigate the magnitude of the delay in a high-current discharge, breakdowns between two plane parallel circular aluminum electrodes in low-pressure stagnant nitrogen gas were observed. The electrodes were 1 in. in diameter and were separated by a 1-cm gap. A thyratron switch connected a charged 1-μf capacitor bank across the electrodes when the thyratron was triggered. The electrode voltage was measured with a voltage probe and discharge current was measured with a Rogowski coil. When the thyratron was triggered, the voltage oscilloscope traces showed that the capacitor voltage was applied across the electrodes with a rise time less than 0.1 μsec. The voltage remained constant until a high current discharge occurred. The discharge produced a sharp decrease in voltage which coincided with the beginning of the damped sinusoidal discharge current signal. The peak value of the discharge current was almost 2000 amp. All the reported measurements were taken with an initial capacitor voltage of 3 kv, the maximum voltage rating of the capacitor bank. Lower voltages did not consistently produce discharges in the pressure range of 0.2 to 1.0 mm at which the measurements were made. The measured value of the d.c. sparking potential in this pressure range was about 350 v. The interval between successive discharges was generally 30 sec. No noticeable changes were produced when this interval was increased to 5 min.

The delays found in 139 measurements at a pressure of 0.2 mm Hg and 60 measurements at 0.7 mm Hg are shown in Fig. 1. The fraction of the delays falling in 40-μsec intervals is plotted. There was considerable scatter in the measured values. Examination of the data shows that half the delays at 0.2 mm Hg were longer than 160 μsec and more than 20% were longer than 480 μsec. At 0.7 mm Hg, the delays were somewhat shorter, half being longer than 80 μsec and 10% longer than 480 μsec. Delays of several milliseconds were occasionally observed. Thus, a considerable fraction of the observed delays were long enough to affect propellant efficiency.

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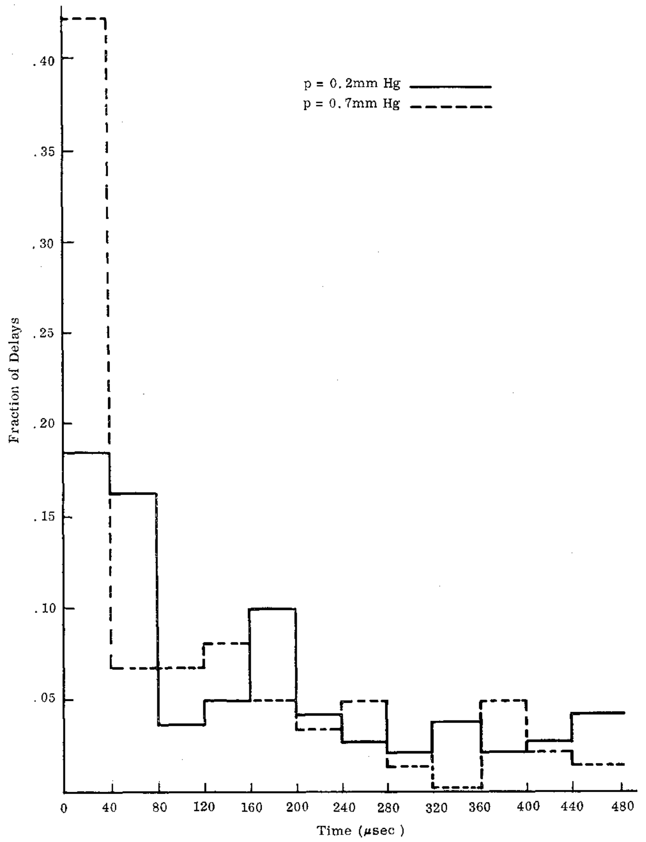

Fig. 1 Fraction of delays occurring in 40-μsec intervals.

It was at first suspected that the delays might be caused by discharge processes that limited current, after initiation of the discharge, to a small value, until some cathode electron emission process capable of supporting high current had time to establish itself. Therefore, a 1-meg resistance was placed in series between the electrodes and the capacitors. Even a small discharge current would produce an observable decrease in electrode voltage because of the voltage drop across the resistor. Thus, if the supposition had been correct, the resistor would have considerably shortened the observed delay in the voltage drop across the electrodes.

However, the results of these tests showed that no significant reduction in delay was produced by the resistor. Thus, the observed delays apparently were caused by statistical lags in the initiation of the discharges rather than lags in current buildup after initiation.

To further test this conclusion an auxiliary voltage supply was connected across the electrodes, so that a d.c. discharge of a few milliamperes was maintained between them. Under this condition, statistical lags were eliminated since a discharge was already in existence when the thyratron switch was fired. As expected, the delays under this condition were all very short, less than 10 μsec, and there was only small scatter in the recorded values.

The choice of aluminum, a metal commonly used for accelerator electrodes, may have reduced statistical lags considerably compared to those that might be found with other metals. Aluminum forms a thin, highly insulating oxide layer on its surface, and positive ions can collect on this layer. The resulting positive charge can produce sufficiently intense electric fields at the electrode surface to cause some cold cathode electron emission, and thus provide free electrons. This phenomenon, the "Malter" or "Paetow" effect (Ref. 3, p. 113), is familiar in Geiger counter work since electrons thus emitted sometimes cause spurious counts for several minutes after a count has been recorded. Therefore, since the interval between successive discharges was 30 sec, electrons emitted by this mechanism from oxide patches on the aluminum may have reduced statistical lags for the discharges following the initial one in a series. It would be useful to investigate whether electrodes constructed of metals that do not form insulating oxide layers, or the use of hydrogen propellant, which reduces oxides, can produce excessively long time delays in accelerators.

References

[1] Gooding, T. J., Hayworth, B. R., and Lovberg, R. H., "Instabilities in a coaxial plasma gun," AIAA J. **1**, 1289–1292 (1963).

[2] Duclos, D. P., Aronowitz, L., Fessenden, F. P., and Carstensen, P. B., "Diagnostic studies of a pinch plasma accelerator," AIAA J. **1**, 2505–2513 (1963).

[3] Meek, J. M. and Craggs, J. D., *Electrical Breakdown of Gases* (Oxford University Press, London, 1953), pp. 111 ff.

Asymptotic Solution of a Toroidal Shell Subjected to Nonsymmetric Loads

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The determination of stresses and strains in a thin shell of revolution subjected to arbitrary loading reduces, according to Novozhilov's theory,[3] to integration of two differential equations in two complex stress function $\bar{U}_k(\Theta)\cos k\phi$ and $\bar{T}_k(\Theta)\cos k\phi$.

These equations, when specialized to a circular toroidal shell, acquire the following form:

$$G_k\left(\frac{\bar{U}_k}{r}\right) + \alpha k^2\left(\sin^2\Theta + \frac{1}{i\lambda^2}\frac{\alpha}{\rho}\right)\bar{T}_k = \frac{R}{\rho}\, g_k(\Theta) \quad \text{(1a)}$$

$$\frac{1}{i\lambda^2}\, G_k(\bar{T}_k) + \bar{T}_k \sin^2\Theta + \frac{1}{\rho r}\,\bar{U}_k = R\rho q_{n,k}(\Theta)\sin\Theta \quad \text{(1b)}$$

where

$$0 < \alpha = r/R < 1 \quad (2)$$

$$\lambda^2 = \frac{r^2}{Rt}\,[12(1-\nu^2)]^{1/2} \gg 1 \quad (3)$$

$$\rho = 1 + \alpha\sin\Theta \quad (4)$$

$$G_k(\ldots) = \frac{\sin^2\Theta}{\rho}\frac{d}{d\Theta}\left[\frac{\rho^2}{\sin\Theta}\frac{d(\ldots)}{d\Theta}\right] - \frac{k^2\alpha^2}{\rho}(\ldots)\sin\Theta \quad (5)$$

$$g_k(\Theta) = \sin^2\Theta(d/d\Theta)[\rho^3(q_{n,k}\cot\Theta - q_{1,k})] + k\alpha q_{2,k}\rho^2\sin^2\Theta \quad (6)$$

$$q_{1,k}(\Theta)\cos k\phi \qquad q_{2,k}(\Theta)\sin k\phi \qquad q_{n,k}(\Theta)\cos_k\phi$$

are the load components in the meridian, circumferential, and normal directions, respectively, ν is Poisson's ratio, and the remaining notation is explained in Fig. 1. The harmonic index k may have an integer value $n = 0, 1, 2, \ldots$ for shells with edges only at $\Theta = \text{const}$, as well as a noninteger one, $k = n(2\pi/\phi_0)$, for shells with edges at $\phi = 0$ and $\phi = \phi_0$ also.

Solutions of Eqs. (1) are known for the axisymmetrical case[3] $k = 0$ and for sinusoidal loading[4] $k = 1$ (see Ref. 2 for bibliography).

Moreover, parts of the toroid sufficiently distant from the top parallel circles, where the inequality

$$\lambda^2\sin^2\Theta \gg 1 \quad (7)$$

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